Assessment of Scale Variable of Log-Gamma Distribution: A Bayesian Technique

Gambo S. A.[1], Hussain Y.[2], Abba S. A.[3], Isa I.[4], Luqman A.[5]

[1]Department of Statistics, Kano State Polytechnic, Kano-Nigeria, 07033154814, salihugabdullahi@yahoo.com
[2]Department of Statistics, Kano University of Science and Technology Wudil, Kano-Nigeria, 08030935350, yhtiga@yahoo.com
[3]Department of Art and Humanitarian Studies, Kano State Polytechnic, Kano-Nigeria, Corresponding Author: 08036611280, shehuabba12@gmail.com,
[4]Department of Art and Humanitarian Studies, Kano State Polytechnic, Kano-Nigeria, 08038493819, ibistatistics@kanopoly.edu.ng
[5]Department of Statistics, Kano State Polytechnic, Kano-Nigeria, 08034706677, abdulrrasheedluqman@gmail.com

Correspondence author: salihugabdullahi@yahoo.com, yhtiga@yahoo.com

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Abstract

The aim of this paper is to estimate scale parameter β of Log Gamma distribution by using inverse Gamma and inverse Chi-Square priors. The measures of loss function; Squared error loss function (SELF) and Quadratic loss function (QLF) are compared through the estimates of the scale parameter β for best results. Wolfram Mathematica 11 was used for the analysis. The results showed that estimates of the scale parameter decrease with increase in sample size tending to the actual value of the scale parameter. This indicates an increase in the estimate of shape parameter under loss functions being considered. However, inverse chi square prior outperforms inverse gamma prior. The posterior risks for QLF are least compared to those under SELF. The Quadratic loss function therefore, appeared to be better than Squared error loss function.

Keywords: *Informative Prior, Posterior, Loss Function and Wolfram Mathematica 11*

1. Introduction

Bayesian method of estimation is being taken into consideration in this research work. This method has numerous merits over the conventional approach. Bayesian inference emerges out of the concept in Bayes' rule which utilizes a prior knowledge about the distribution of a given parameter β as well as actual observation of that distribution to come up with a posterior distribution that helps us upgrade our previous understanding. Ahmad *et al.* (2016) studied the Bayesian estimates of parameters of length biased Nakagami distribution by using different loss functions and different priors. The results showed that the length biased Nakagami distribution gives better output compared to Nakagami and Rayleigh distributions. Terna and Unna (2018) estimate the shape parameter of Weibull Frechet distribution by using Bayesian approach. Two non-informative priors and three different loss functions were involved. The corresponding posterior distribution was drive for the shape parameter of the Weibull Frechet distribution. The bays' estimators and their corresponding bays' risk were also drive using the three selected loss functions. A comprehensive simulation ideology was used to compare the performance of the bays' estimates evaluated in the study in order to find out the combination of loss function and prior with a minimum bays' risk. Hence, the study concluded that Quadratic loss function under either of the two non-informative prior is the best when estimating the parameters in the study.

Dikko and Mathew (2018) used a Bayesian approach to estimate the scale parameter of a Frechet distribution the posterior distribution was drive using two non-informative priors Uniform and Jeffreys priors under four loss functions (Squared error, Weighted balance, Quadratic and Precautionary loss functions). The study revealed that Weighted balance loss function when used with uniform prior provides the least posterior risk. Loaiy and Huda (2019) in their study obtained the bays' estimators for the scale and shape parameters under Entropy loss function assuming exponential and Gamma priors for the scale and shape parameters respectively. The study revealed that the bays' estimates under Entropy loss function performs better than the other estimates in all cases. Innocent *et al.* (2020) uses a 3-parameter distribution; Weibull- Lindley distribution and the priors involved are Gamma, Jeffrey's and Uniform priors under Quadratic loss function, squared error loss function and Precautionary loss function. The result revealed that Quadratic Loss function (QLF) provides estimates with the least MSE's under the prior distributions involved.

Bayes estimates decrease with an increase in the sample size and approaches the actual value of scale parameter. Hence the consistence of the Bayes estimate was proved matched with the theory of Bayesian analysis (Zaka and Akhter, 2014 & feroze, 2012). Keeping the scale parameter consisted, the posterior risk decreases and the shape parameter increases under all priors and loss functions, and this is similar with the outcome obtained by (Feroze and Yaseen, 2015). In this research, the scale parameter of Log gamma distribution was to estimate by using inverse gamma and inverse chi square priors under the Quadratic and Squared error loss functions respectively.

2. Material and Methods

2.1. Posterior distribution using inverse Gamma priors

$$p(\beta|x)=\frac{p(x|\beta)\times p(\beta)}{\int_{-\infty}^{\infty}p(x|\beta)\times p(\beta)d\beta} \tag{1}$$

The probability density function of log gamma distribution is given by:

$$f(x;\lambda,\beta)=\frac{(\ln x)^{\lambda-1}}{\Gamma(\lambda)\beta^{\lambda}}x^{-1}e^{\frac{-\ln x}{\beta}} \qquad 0 < x < \infty. \tag{2}$$

Where, β is the scale parameter and λ is the shape parameter

The likelihood function of log gamma distribution can be obtained by:

$$p(x|\beta)=\prod_{i=1}^{n}\{f(x;\lambda.\beta)\}$$

$$p(x|\beta)=\frac{\prod_{i=1}^{n}(\ln x_i)^{\lambda-1}}{(\Gamma\lambda)^n(\beta)^{n\lambda}}\prod_{i=1}^{n}x^{-1}e^{\frac{-\sum_{i=1}^{n}\ln x_i}{\beta}} \tag{3}$$

Inverse Gamma Prior is given by

$$p(\beta)=\frac{m^a}{\Gamma(a)}\beta^{-a-1}e^{\left(-\frac{m}{\beta}\right)}, \qquad \text{where } m,a,\beta>0 \tag{4}$$

Substituting equations (3) and (4) into equation (1) will yield

$$p(\beta|x)=\frac{\beta^{-n\lambda-a-1}e^{-\left(m/\beta+1/\beta\sum_{i=1}^{n}\ln x_i\right)}}{\int_0^\infty \beta^{-n\lambda-a-1}e^{-\left(m/\beta+1/\beta\sum_{i=1}^{n}\ln x_i\right)}d\beta} \tag{5}$$

Let

$$c=\int_0^\infty \beta^{-n\lambda-a-1}e^{-\left(m/\beta+1/\beta\sum_{i=1}^{n}\ln x_i\right)}d\beta \tag{6}$$

and

$$s=\left(m+\sum_{i=1}^{n}\ln x_i\right)\frac{1}{\beta} \tag{7}$$

$$ds=-\left(m+\sum_{i=1}^{n}\ln x_i\right)\frac{1}{\beta^2}d\beta$$

Therefore,

$$\beta=\left(m+\sum_{i=1}^{n}\ln x_i\right)\frac{1}{s} \tag{8}$$

$$d\beta=\frac{\left[\left(m+\sum_{i=1}^{n}\ln x_i\right)\frac{1}{s}\right]^2}{\left(m+\sum_{i=1}^{n}\ln x_i\right)}ds$$

By substitution, equation (6) becomes

$$c=\int_0^\infty\left[\left(m+\sum_{i=1}^{n}\ln x_i\right)\frac{1}{s}\right]^{-n\lambda-a-1}e^{-u}\frac{\left[\left(m+\sum_{i=1}^{n}\ln x_i\right)\frac{1}{s}\right]^2}{\left(m+\sum_{i=1}^{n}\ln x_i\right)}ds$$

Hence, equation (5) becomes

$$p(\beta|x)=\frac{\beta^{-n\lambda-a-1}e^{-\left(m/\beta+1/\beta\sum_{i=1}^{n}\ln x_i\right)}\left(m+\sum_{i=1}^{n}\ln x_i\right)^{n\lambda+a}}{\Gamma(n\lambda+a)} \tag{9}$$

2.2. Posterior distribution using inverse Chi-square prior

$$p(\beta) = \frac{2^{-\frac{v}{2}}}{\Gamma\left(\frac{v}{2}\right)} \beta^{-\frac{v}{2}-1} e^{-\left(\frac{1}{2\beta}\right)}, \qquad \text{where } v, \beta > 0 \tag{10}$$

Substituting equations (3) and (10) into equation (1) will yield

$$p(\beta|x) = \frac{\beta^{-n\lambda - \frac{v}{2}-1} e^{-\left(\frac{1}{2\beta} + \frac{1}{\beta}\sum_{i=1}^{n} \ln x_i\right)}}{\int_0^\infty \beta^{-n\lambda - \frac{v}{2}-1} e^{-\left(\frac{1}{2\beta} + \frac{1}{\beta}\sum_{i=1}^{n} \ln x_i\right)} d\beta} \tag{11}$$

Let

$$\mathbf{B} = \int_0^\infty \beta^{-n\lambda - \frac{v}{2}-1} e^{-\left(\frac{1}{2\beta} + \frac{1}{\beta}\sum_{i=1}^{n} \ln x_i\right)} d\beta \tag{12}$$

and

$$u = \left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)\frac{1}{\beta} \tag{13}$$

$$du = -\left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)\frac{1}{\beta^2} d\beta$$

Therefore

$$\beta = \left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)\frac{1}{u} \tag{14}$$

$$d\beta = \frac{\beta^2 \, du}{\left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)} = \frac{\left[\left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)\frac{1}{u}\right]^2}{\left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)} du \tag{15}$$

By substitution, equation 12 becomes

$$B = \int_0^\infty \left[\left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)\frac{1}{u}\right]^{-n\lambda - \frac{v}{2}-1} e^{-u} \frac{\left[\left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)\frac{1}{u}\right]^2}{\left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)} du$$

Hence, equation 11 becomes

$$p(\beta|x) = \frac{\beta^{-n\lambda - \frac{v}{2}-1} e^{-\left(\frac{1}{2\beta} + \frac{1}{\beta}\sum_{i=1}^{n} \ln x_i\right)} \left(\frac{1}{2} + \sum_{i=1}^{n} \ln x_i\right)^{n\lambda + \frac{v}{2}}}{\Gamma(n\lambda + \frac{v}{2})} \tag{16}$$

2.3. Bayesian Estimator under SELF using Inverse Gamma prior

$$L(\beta, \beta_{SELF}) = (\beta - \beta_{SELF})^2, \qquad \beta_{SELF} = E(\beta) \quad \text{and} \quad E(\beta) = \int \beta p(\beta|x) d\beta$$

From equation 10

$$E(\beta)=\int_0^\infty \beta \frac{\beta^{-n\lambda-a-1} e^{-\left(\frac{m}{\beta}+\frac{1}{\beta}\sum_{i=1}^{n}\ln x_i\right)}\left(m+\sum_{i=1}^{n}\ln x_i\right)^{n\lambda+a}}{\Gamma(n\lambda+a)} d\beta$$

Hence $$\hat{\beta}_{SELF}=\frac{\left(m+\sum_{i=1}^{n}\ln x_i\right)}{(n\lambda+a-1)} \tag{17}$$

2.4.Bayes estimate under SELF using inverse chi-square prior

$$E(\beta)=\int_0^\infty \beta \frac{\beta^{-n\lambda-\frac{v}{2}-1} e^{-\left(\frac{1}{2\beta}+\frac{1}{\beta}\sum_{i=1}^{n}\ln x_i\right)}\left(\frac{1}{2}+\sum_{i=1}^{n}\ln x_i\right)^{n\lambda+\frac{v}{2}}}{\Gamma(n\lambda+\frac{v}{2})} d\beta$$

Hence $$\beta_{SELF}=\frac{\left(\frac{1}{2}+\sum_{i=1}^{n}\ln x_i\right)}{(n\lambda+\frac{v}{2}-1)}$$

(18)

2.5. Bayes estimator under QLF using inverse gamma prior

$$L(\beta,\beta_{QLF})=\left(\frac{\beta-\beta_{SELF}}{\beta}\right),\qquad \beta_{QLF}=\frac{E(\beta^{-1})}{E(\beta^{-2})}\quad \text{and}\quad E(\beta^{-1})=\int \beta^{-1}p(\beta|x)d\beta$$

$$E(\beta^{-1})=\int_0^\infty \beta^{-1} \frac{\beta^{-n\lambda-a-1} e^{-\left(\frac{m}{\beta}+\frac{1}{\beta}\sum_{i=1}^{n}\ln x_i\right)}\left(m+\sum_{i=1}^{n}\ln x_i\right)^{n\lambda+a}}{\Gamma(n\lambda+a)} d\beta=\frac{n\lambda+a}{\left(m+\sum_{i=1}^{n}\ln x_i\right)}$$

Also $E(\beta^{-2})=\int \beta^{-2}p(\beta|x)d\beta$

$$E(\beta^{-2})=\int_0^\infty \beta^{-2} \frac{\beta^{-n\lambda-a-1} e^{-\left(\frac{m}{\beta}+\frac{1}{\beta}\sum_{i=1}^{n}\ln x_i\right)}\left(m+\sum_{i=1}^{n}\ln x_i\right)^{n\lambda+a}}{\Gamma(n\lambda+a)} d\beta=\frac{(n\lambda+a+1)(n\lambda+a)}{\left(m+\sum_{i=1}^{n}\ln x_i\right)^2}$$

Hence

$$\beta_{QLF}=\left[\frac{\frac{(n\lambda+a)}{\left(m+\sum_{i=1}^{n}\ln x_i\right)}}{\frac{(n\lambda+a+1)(n\lambda+a)}{\left(m+\sum_{i=1}^{n}\ln x_i\right)^2}}\right]=\frac{\left(m+\sum_{i=1}^{n}\ln x_i\right)}{(n\lambda+a+1)} \tag{19}$$

2.6. Bayes estimator under QLF using inverse chi-square prior

$$\beta_{QLF} = \frac{E(\beta^{-1})}{E(\beta^{-2})}$$

And
$$E(\beta^{-1}) = \int \beta^{-1} p(\beta|x) d\beta$$

$$E(\beta^{-1}) = \int_0^\infty \beta^{-1} \frac{\beta^{-n\lambda - v/2 - 1} e^{-\left(1/2\beta + 1/\beta \sum_{i=1}^n \ln x_i\right)} \left(\frac{1}{2} + \sum_{i=1}^n \ln x_i\right)^{n\lambda + v/2}}{\Gamma(n\lambda + v/2)} d\beta = \frac{n\lambda + v/2}{\left(1/2 + \sum_{i=1}^n \ln x_i\right)}$$

Also $\quad E(\beta^{-2}) = \int \beta^{-2} p(\beta|x) d\beta$

$$E(\beta^{-2}) = \int_0^\infty \beta^{-2} \frac{\beta^{-n\lambda - v/2 - 1} e^{-\left(1/2\beta + 1/\beta \sum_{i=1}^n \ln x_i\right)} \left(\frac{1}{2} + \sum_{i=1}^n \ln x_i\right)^{n\lambda + v/2}}{\Gamma(n\lambda + v/2)} d\beta = \frac{(n\lambda + v/2 + 1)(n\lambda + v/2)}{\left(\frac{1}{2} + \sum_{i=1}^n \ln x_i\right)^2}$$

Hence

$$\beta_{QLF} = \left[\frac{(n\lambda + v/2)}{\left(1/2 + \sum_{i=1}^n \ln x_i\right)} \Bigg/ \frac{(n\lambda + v/2 + 1)(n\lambda + v/2)}{\left(\frac{1}{2} + \sum_{i=1}^n \ln x_i\right)^2} \right] = \frac{\left(1/2 + \sum_{i=1}^n \ln x_i\right)}{\left(n\lambda + v/2 + 1\right)} \tag{20}$$

2.7. Posterior risk under SELF

$$\rho(\beta_{SELF}) = E(\beta^2) - \{E(\beta)\}^2$$

2.7.1 Posterior risk using Gamma prior

$$\rho_{SELF} = \frac{\left(m + \sum_{i=1}^n \ln x_i\right)^2}{(n\lambda + a - 1)(n\lambda + a - 2)} - \left(\frac{\left(m + \sum_{i=1}^n \ln x_i\right)}{(n\lambda + a - 1)}\right)^2 = \frac{\left(m + \sum_{i=1}^n \ln x_i\right)^2}{(n\lambda + a - 1)^2 (n\lambda + a - 2)}$$

2.7.2 Posterior risk using inverse chi square prior

$$\rho_{SELF} = \frac{\left(\frac{1}{2} + \sum_{i=1}^n \ln x_i\right)^2}{(n\lambda + v/2 - 1)(n\lambda + v/2 - 2)} - \left(\frac{\left(\frac{1}{2} + \sum_{i=1}^n \ln x_i\right)}{(n\lambda + v/2 - 1)}\right)^2 = \frac{\left(\frac{1}{2} + \sum_{i=1}^n \ln x_i\right)^2}{(n\lambda + v/2 - 1)^2 (n\lambda + v/2 - 2)}$$

2.8. Posterior risk under QLF

$$\rho(\beta_{QLF}) = 1 - \frac{\{E(\beta^{-1})\}^2}{E(\beta^{-2})}$$

2.8.1 Posterior risk using inverse gamma prior

$$\rho_{QLF} = 1 - \left\{ \left(\frac{(n\lambda + a)}{\left(m + \sum_{i=1}^{n} \ln x_i \right)} \right)^2 \div \frac{(n\lambda + a + 1)(n\lambda + a)}{\left(m + \sum_{i=1}^{n} \ln x_i \right)^2} \right\} = \frac{1}{n\lambda + a + 1}$$

2.8.2 Posterior risk using inverse chi square prior

$$\rho_{QLF} = 1 - \left\{ \left(\frac{\left(n\lambda + v\, {}^{v}\!/_{2}\right)}{\left({}^{1}\!/_{2} + \sum_{i=1}^{n} \ln x_i \right)} \right)^2 \div \frac{\left(n\lambda + {}^{v}\!/_{2} + 1\right)\left(n\lambda + {}^{v}\!/_{2}\right)}{\left({}^{1}\!/_{2} + \sum_{i=1}^{n} \ln x_i \right)^2} \right\} = \frac{1}{\left(n\lambda + {}^{v}\!/_{2} + 1\right)}$$

3. Analysis

The scale parameter of log gamma distribution was estimated under Bayesian framework, in this section, Monte Carlo simulation (MCMC) is carried out to obtain the numerical value of the Bayes estimate and posterior risk with sample sizes (n = 5, 20 and 50). In inverse gamma prior, (m, a) = (0.5, 1.5) and (1, 1) are values of the hyper parameters used. For inverse chi square prior, the hyper parameter v takes the values 0.5 and 1.5, using the values of shape and scale parameter:

$$(\alpha, \beta) \in \{(0.5,0.5), (0.5,1), (0.5,1.5), (1,0.5), (1.5,0.5)\}$$

The estimated parameter β under specified actual values shape and scale parameter (λ, β), and different value of hyper parameters (m, a) for inverse gamma and for inverse chi square (v), are shown on the tables below, under different priors and loss functions.

Table ١: Estimates of β and Risk under SELF using Inverse Gamma prior (0.5, 1.5)

m = 0.5 and a = 1.5					
N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
5	0.583623	1.00031	1.41821	0.548464	0.528996
	0.206106	0.636373	1.32547	0.076208	0.044148
20	0.524544	1.00304	1.4773	0.514145	0.507866
	0.031391	0.115214	0.250885	0.014171	0.009014
50	0.511862	0.999912	1.48735	0.504482	0.503648
	0.011086	0.042398	0.09382	0.005241	0.00345

Table ٢: Estimates of β and Risk under SELF using Inverse Gamma prior (1,1)

m = 1 and a = 1					
N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
5	0.58578	1.00302	1.41126	0.546613	0.531913
	0.206803	0.644219	1.29829	0.0757508	0.0446906
20	0.52466	0.996265	1.46984	0.511241	0.509691

	0.0313543	0.114052	0.24912	0.0140242	0.00907758
	0.507491	0.996783	1.48906	0.50606	0.502663
50	0.0108983	0.0420964	0.0940776	0.00527248	0.00343664

Table ٣: Estimates of β and Risk under QLF using Inverse Gamma prior (0.5, 1.5)

m = 0.5 and a = 1.5					
N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
	0.351175	0.598303	0.849588	0.399296	0.425448
5	0.2	0.2	0.2	0.133333	0.1
	0.438859	0.838541	1.23856	0.466928	0.47735
20	0.08	0.08	0.08	0.0444444	0.0307692
	0.474004	0.928107	1.37994	0.48513	0.49038
50	0.0363636	0.0363636	0.0363636	0.0190476	0.0129032

Table ٤: Estimates of β and Risk under QLF using Inverse Gamma prior (1, 1)

m = 1 and a = 1					
N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
	0.350399	0.599512	0.849048	0.403354	0.425747
5	0.2	0.2	0.2	0.133333	0.1
	0.441245	0.838993	1.23846	0.466679	0.47636
20	0.08	0.08	0.08	0.0444444	0.0307692
	0.473473	0.929581	1.38283	0.48672	0.490318
50	0.0363636	0.0363636	0.0363636	0.0190476	0.0129032

Table ٥: Estimates of β and Risk under SELF using Inverse Chi square prior (V = 0.5)

N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
	0.995077	1.71861	2.42647	0.710195	0.630849
5	9.29869	29.5747	60.1325	0.267994	0.0974525
	0.592833	1.13868	1.6801	0.544886	0.52987
20	0.0548621	0.204099	0.445128	0.0184359	0.0108005
	0.536827	1.05025	1.56593	0.518855	0.510887
50	0.0136854	0.0524768	0.116734	0.00586399	0.00368508

Table ٦: Estimates of β and Risk under SELF using Inverse Chi square prior (V = 1.5)

N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
	0.772747	1.33228	1.89917	0.635546	0.589214
5	-2.86833	-9.11283	-18.7152	0.204149	0.0807202
20	0.563109	1.06989	1.5916	0.531907	0.521022

	0.0473119	0.172227	0.381938	0.0171408	0.0102675
50	0.525267	1.0317	1.54029	0.513635	0.508511
	0.0128524	0.0496912	0.110915	0.00569308	0.00362515

Table ٧: Estimates of β and Risk under QLF using Inverse Chi square prior (V = 0.5)

N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
5	0.777617	1.31973	1.88289	0.630672	0.588436
	-2.91227	-8.86796	-18.6118	0.201394	0.0804884
20	0.562687	1.07808	1.59438	0.530682	0.520414
	0.0473316	0.174819	0.383587	0.0170813	0.0102467
50	0.524947	1.0333	1.53551	0.510894	0.509592
	0.012844	0.0498732	0.110045	0.00563079	0.00364042

Table ٨: Estimates of β and Risk under QLF using Inverse Chi square prior (V = 1.5)

N	α = 0.5 β = 0.5	α = 0.5 β = 1	α = 0.5 β = 1.5	α = 1 β = 0.5	α = 1.5 β = 0.5
5	0.410983	0.707766	1.004	0.444051	0.461136
	0.235294	0.235294	0.235294	0.148148	0.108108
20	0.468479	0.898738	1.32276	0.482734	0.487657
	0.0851064	0.0851064	0.0851064	0.045977	0.0314961
50	0.484461	0.953276	1.41889	0.491984	0.495538
	0.0373832	0.0373832	0.0373832	0.0193237	0.0130293

4. Results and Discussion of Findings

It is observed from the analysis that the Bayes estimate increases as the sample size increases tending to the actual value of scale parameter for all priors as well as the loss functions. For inverse gamma prior using SELF, the Bayes estimate and posterior risk increase when the shape parameter increases and also when it is fixed. The posterior risk decreases with increase in sample size for SELF, QLF and both priors. The posterior risks remain constant when the shape parameter is fixed, and also decreases with increase in the shape parameter when the scale parameter is fixed. For inverse Chi-square prior with hyper parameter (v), the posterior risk decreases with increase in the values of hyper parameter, this property holds for QLF. However, the estimates under QLF produces least amount of posterior risk under both inverse gamma and inverse chi square prior respectively, but the QLF under inverse chi-square prior produce least posterior risk than SELF of both priors including QLF under inverse gamma prior.

5. Conclusion

The aim of this research work was to estimate the scale parameter of Log gamma distribution by using inverse gamma and inverse chi square priors under the Quadratic and Squared error loss function respectively. However, Bayes estimate decreases as the size of the sample increases tending to the actual value of scale parameter. Hence the consistence of the Bayes estimate was proved matched with the theory of Bayesian analysis. Keeping the scale parameter consisted, the posterior risk decreases and the shape parameter increases under all priors and loss functions. However, inverse chi square prior performs better than inverse gamma prior, similarly, the posterior risk for QLF is least compared with that under SELF. Hence Quadratic loss function appeared to be best when inverse chi square prior is used.

Author's contribution

1. The Posterior distributions using both inverse Gamma and inverse Chi-Square priors are derived by Gambo S. A.
2. The Bayes estimators under SELF using both inverse Gamma and inverse Chi-square are derived by Hussain Y.
3. The Bayes estimators under QLF using both inverse Gamma and inverse Chi-square are derived by Abba S. A.
4. The Posterior risk under SELF and QLF using both inverse Gamma and inverse Chi-square are derived by Isa I.
5. The analysis, conclusion and references are prepared by Luqman A.

Area of Further Research

In step with this research work, further research can be done towards Posterior analysis of log gamma distribution using different prior and loss function, because, better combination of priors & loss functions may be outside the subjected one used for this study.

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